MC Part – hand in this sheet along with your solution to the previous problems.

Student Number: _____

MC Problem 1. Let P and Q be logical propositions.
Which of the following logical propositions is equivalent to

$$(\neg Q \wedge P) \wedge \neg(\neg P \vee Q)\,?$$

Answer Options (mark only one):

○ $P \vee \neg Q$	○ $P \wedge Q$	○ $\neg P \wedge Q$	○ $\neg P \wedge \neg Q$
● $P \wedge \neg Q$	○ $\neg P \vee Q$	○ $P \Rightarrow Q$	○ $Q \Rightarrow P$

MC Problem 2. What is the polar form of the complex number $z = 1 - i$?

Answer Options (mark only one):

● $\sqrt{2}\,e^{-i\pi/4}$	○ $2e^{-i\pi/4}$	○ $\sqrt{2}\,e^{i\pi/4}$	○ $2e^{i\pi/4}$
○ $e^{-i\pi/2}$	○ $\sqrt{2}\,e^{-i\pi/2}$	○ $\sqrt{2}\,e^{i\pi/2}$	○ $e^{-i\pi/4}$

MC Problem 3. Which of the below polynomials is the result of the division

$$\frac{2Z^3 + Z^2 - 12Z + 9}{2Z - 3}\,?$$

Answer Options (mark only one):

○ $Z^2 + 2Z - 1$	○ $Z^2 - Z + 3$	● $Z^2 + 2Z - 3$	○ $Z^2 - 2Z + 3$
○ $Z^2 + Z - 3$	○ $Z^2 - 3Z + 1$	○ $Z^2 + 3Z - 1$	○ $Z^2 - Z - 3$

MC Problem 4. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 & 3 \end{bmatrix}.$$

Choose the result of the calculation $\mathbf{A}^{-1} \cdot \mathbf{b}^T$.

Answer Options (mark only one):

● $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$	○ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	○ $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$	○ $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$
○ $\begin{bmatrix} 1 & -2 \end{bmatrix}$	○ $\begin{bmatrix} 2 & 1 \end{bmatrix}$	○ $\begin{bmatrix} -1 & -2 \end{bmatrix}$	○ $\begin{bmatrix} 3 & 1 \end{bmatrix}$

MC Problem 5. Which of the following numbers is an eigenvalue of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}\,?$$

Answer Options (mark only one):

○ -4	● -1	○ 0	○ 1
○ -2	○ 3	○ 4	○ 6

MC Problem 6. Let $f : \mathbb{N} \to \mathbb{R}^2$ be a recursively defined function given by

$$f(1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad f(n) = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} f(n-1),\ n \geq 2.$$

Which of the below vectors equals $f(3)$?

Answer Options (mark only one):

○ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$	○ $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$	○ $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$	○ $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$
○ $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$	○ $\begin{bmatrix} 0 \\ -4 \end{bmatrix}$	○ $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$	◯ $\begin{bmatrix} 1 \\ -7 \end{bmatrix}$

MC Problem 7. Let V be a subspace of $\mathbb{R}^3$ with ordered basis

$$b = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right).$$

Which of the following vectors in $\mathbb{R}^3$ does not belong to V?

Answer Options (mark only one):

○ $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$	○ $\begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$	○ $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$	○ $\begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$
○ $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$	○ $\begin{bmatrix} 5 \\ -1 \\ 5 \end{bmatrix}$	○ $\begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}$	◯ $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

MC Problem 8. Let $a, b \in \mathbb{R}$ be constants and let $q : \mathbb{R} \to \mathbb{R}$ be a function.
Consider the differential equation

$$f''(t) + af'(t) + bf(t) = q(t)\,.$$

For which values of a, b and $q(t)$ is the function

$$f(t) = e^t \cos(2t) + 3e^t \sin(2t) + 5t$$

a solution?

Answer Options (mark only one):

○ $a = -2, \quad b = 5, \quad q(t) = 10t - 25$	○ $a = -2, \quad b = 2, \quad q(t) = 10t - 10$
○ $a = 2, \quad b = 5, \quad q(t) = 25t + 10$	○ $a = -1, \quad b = 5, \quad q(t) = 25t$
○ $a = -2, \quad b = 4, \quad q(t) = 20t - 10$	○ $a = 2, \quad b = 5, \quad q(t) = 25t - 10$
◯ $a = -2, \quad b = 5, \quad q(t) = 25t - 10$	○ $a = -2, \quad b = 3, \quad q(t) = 15t - 10$

END OF EXAM